IMPLICATION OF ECONOMETRIC SPECIFICATION ON ECONOMIC ELASTICITY ESTIMATES : A Case of Generalized Leontief Profit Function

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Abstrak

Pemilihan suatu bentuk fungsi untuk menjelaskan kaitan antara satu peubah dengan peubah lain sering ditentukan oleh penguasaan penganalisa terhadap teknik pendugaan ekonometrik kaitan tersebut. Makalah ini menganalisis elastisitas hasil dari permintaan masukan usahatani padi di Jawa dan menyimpulkan bahwa elastisitas-elastisitas ini tidak bebas dari bentuk fungsi dan perumusan ekonometriknya. Malahan dari suatu bentuk fungsi dapat diperoleh elastisitas-elastisitas yang berbeda apabila perumusan ekonometriknya berbeda. Makalah ini menyarankan agar perumusan ekonometrik dilakukan dengan memanfaatkan seluruh informasi yang ada pada data.

INTRODUCTION

Economic theory provides guidance on relationship between economic variables in general, but it does not say anything about types of functional forms of the relationship that are needed in empirical analysis. Therefore, in an empirical study, an analyst should be aware that a choice of functional form should be guided by not only the economic or statistical theory but also by his intuition, judgement and validity of parameters or elasticities interpretation to the issues at hand. The former body of theory may indicate a situation of elasticity measures changing with price and quantity due to choice of forms. Or as is often the case, statistical consideration such as the distribution of the error term may indicate one functional form is more appropriate than others. This paper is intended to show that in a given data set a functional form could be estimated with several econometric specifications that result in different magnitude of economic elasticities. This is applied to Generalized Leontief Profit Function.

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METHODOLOGY

Data

The research used data from Struktur Ongkos Usaha Tani Padi published by BPS dated from 1976 to 1989, in Java for three provinces, i.e.; Jawa Barat, Jawa Tengah and Jawa Timur. Labor wage rate is obtained from Bagian Statistik Harga Konsumen dan Harga Pedagang Besar dan Bagian Statistik Keuangan dan Harga Produsen, BPS. The short paper only tries the exercise on paddy crop, because it is a crop that possesses a more reliable data.

Variables of interest are : 1) variable profit defined as value of production minus fertilizer and labor expenses, 2) quantity produced, 3) price of rice as value of production divided by quantity, 4) quantity of fertilizer, 5) price of fertilizer as fertilizer expense divided by its quantity, 6) labor average wage rate for hoeing, planting, and weeding, 7) labor use as labor expense divided by average wage rate. All variables are in per hectare basis.

Functional Form

Ideally, statistical theory suggests functional forms, but there are many numeric functions satisfying the requisite theory. In the literature, for example, we can find many profit function approximation depending upon its flexibility in terms of whether the form can provide either second-order numerical or second-order differential approximation to any functional form[1]. Generalized Leontief form belongs to this group and is formulated as :

$$\begin{aligned}\pi\,(p,w_1,w_2,z) = & -a_o p - a_1 w_1 - a_2 w_2 - 2a_{o1} p^{1/2}\, w_1{}^{1/2} \\ & - 2a_{o2}\, p^{1/2}\, w_2{}^{1/2} - 2a_{12} w_1{}^{1/2}\, w_2{}^{1/2} \\ & - a_{oz}\, p\, z - a_{1z}\, w_1\, z - a_{2z}\, w_2\, z \qquad (1)\end{aligned}$$

where: p is output price, w is input price and z is fixed input, and a's are parameters. Several properties of profit function are (Chambers, 1988) :

P1. $\pi\,(p, w, z) \geqslant 0$;

P2. If $p^1 \quad p^2$, then $\pi\,(p^1,w,z) \geqslant \pi\,(p^2,w,z)$ (non decreasing in p);

P3. If $w^1 \geqslant w^2$, then $\pi\,(p,w_1) \leqslant \pi\,(p,w^2)$ (non decreasing in w);

P4. $\pi\,(p,w,z)$ is convex and continuous in (p,w); and

P5. $\pi\,(tp, tw, z) = t\ \pi\,(p,w,z)$, $t \geqslant 0$ (positive linear homogenity).

P6. If π is differentiable in p and w, the unique profit-maximizing supply and derived-demand for inputs are.

$$y(p,w,z) = \frac{\delta \pi (p,w,z)}{\delta p} \text{ and } x_i(p,w,z) =$$

$$- \frac{\delta \pi (p,w,z)}{\delta w_i} \text{ for any i}$$

where: y(p,w,z) and x_i(p,w,z) are the respective profit-maximizing quantities (Hotelling's lemma).

By applying Hotelling's lemma to equation (1), we will get output supply function as :

$$y(p,w_1,w_2,z) = -a_o - a_{o1}(\frac{w_1}{p})^{1/2} - a_{o2}(\frac{w_2}{p})^{1/2} - a_{0z}\, z \text{ .... (2)}$$

and input demand function as :

$$x_i(p,w_1,w_2,z) = a_i + a_{oi}(\frac{p}{w_i})^{1/2} + a_{ij}(\frac{w_j}{w_i})^{1/2} + a_{iz}\, z \text{ .... (3)}$$

All n derived equations can be estimated simultaneously but the profit function (1) is not linearly independent since it is the linear combination ($\Sigma\ y_i\ P_i$) of the individual equations, where y_i is derived function and P_i is its price. However, the homogeneity constraint is not testable since e_{ij} is estimated residually for each equation (Bapna, *et al.*, 1984). But estimating profit function alone is rarely done since it uses up more degrees of freedom and is subject to multi-collinearity. Equation (2) and (3), each has 4 parameters and also is linear in parameters. If symmetry restriction holds then equations (2) and (3) should be estimated simultaneously with $a_{ij} = a_{ji}$. By doing so, all elasticities can also be derived.

In this paper we propose some alternative specifications as contained in Table A1 of Appendix. Any specification is theoretically plausible, and all restrictions are still held. Specification I and III use up more degrees of freedom than specification II because they consist of more arguments in the right hand side of equal signs, especially in the profit functions. These specification are then estimated using available data on paddy. Estimation is done by maximum likelihood techniques. Results of estimation are summarized in Table 1.

Tabel 1. Yield and input demand elasticities from Generalized Leontief Profit Function.

Specification	Rice price	Fertilizer price	Labor wage
Yield Elasticities w.r.t.			
I	0.027	−0.008	−0.018
II	0.025	−0.009	−0.016
III	0.031	−0.009	−0.022
Fertilizer demand elasticities w.r.t.			
I	0.146	−0.246	0.100
II	0.173	−0.166	−0.007
III	0.160	−0.240	0.080
Labor demand elasticities w.r.t.			
I	0.155	0.049	−0.204
II	0.142	−0.003	−0.138
III	0.183	0.039	−0.222

DISCUSSION

As we can see from Table 1, even though other specifications are basically derived from equation (1) with imposition of symmetry restriction in the estimation, calculated elasticities for a given exogeneous variable are different in magnitude. For this reason, it is not surprising to see that different analyst might have different set of estimates for the same variable. This in turn will imply that they could draw different conclusion and policy recommendation (see Ellis, 1988 and KMEW, 1990). Properties P2 and P3 above imply that own price elasticities for output supply should have positive signs and for input demand negative signs. We also expect that input demand elasticities should have positive signs with respect to output price.

Specifications I, II and III give all signs we expected. Own price elasticities for yield and input demand are positive and negative, respectively.

It appears that overall specification III gives higher absolute elasticities followed by specification I, and the last is specification II. This is true for yield elasticities and labor demand elasticities.

In terms of magnitude, specifications I, II, and III produce comparable results of yield elasticities ranging from 0.015 to 0.031. For fertilizer, own-price elasticities of demand derived from specifications I and III (that is −0.246 and −0.240, respectively) are quite similar and about 50 percent higher than that from specifications II which is −0.166. The same is true for own-price elasticities of demand for labor, in which specification I and III are quite close valuing at −0.204 and −0.222, respectively, but about 50 percent higher than that from specification II, which is −0.138.

In terms of cross-price elasticities among inputs, it appears that specification I and III give the same positive signs, meaning that the input are substitutes, whereas specification II yields in negative signs, implying inputs are complements. This suggests that cross-price elasticities are not invariant to the deletion of equation in the system.

The magnitude of yield elasticities with respect to fertilizer price are almost equal in all specifications ranging from -0.008 to -0.009.

Having compared estimated results, it seems that in order to preserve and utilize all information and data available here, specification I would be a better one to use among the alternative specifications above.

CONCLUSION

The paper had shown that different econometric specification will produce different elasticity estimates given a data set under underlying a functional form. Therefore, it is no surprise to find that an analyst might prescribe different sets of policy recommendation from others simply because parameters that he used are calculated from different econometric specification from the others'. Although economic theory doest not say much about the form of economic relationship, empirical analysis should be guided by the theory, judgement and knowledge of the analyst.

Since we are interested in theoretical consistency as much as possible, it appears that specification I will probably be the most appropriate one due to the fact that we are able to utilize all information available by estimating complete system of equations (profit function, output supply, and input demand formulations), which provides the possibility of incorporating necessary restrictions to any profit function as well as output supply and input demand. Moreover, deletion of equation in the system should not be decided a priori.

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Note

[1] Suppose we have a general linear form function

$$h(z) = \sum_{i=1}^{k} \alpha_i b_i(z) \quad \text{.......} \quad (4)$$

where each bi(z) is a known twice-continuously differentiable, numeric function of z, and each α_i is a parameter (typically to be estimated). The expression in (4) can approximate (under relatively weak conditions) any arbitrary twice-continuously differentiable function in the sense that the parameters of (4) can be chosen to ensure that, for any arbitrary h*(z).

$$\begin{aligned} h^*(z^o) &= h(z^o) \\ \nabla_z h^*(z^o) &= \nabla_z h(z) \quad \text{.......} \quad (5) \\ \nabla_{zz} h^*(z^o) &= \nabla_{zz} h(z^o) \end{aligned}$$

where ∇_z denotes gradient of the function it notifies and ∇_{zz} denotes the Hessian matrix (the matrix of second derivatives). That is, the parameters of (4) can be chosen such that its function value, gradient, and Hessian equal the corresponding magnitutes for any arbitrary h*(z) at zo. Functions satisfying (5) are called second-order differential approximations.

Expression (4) also can be formulated to represent a Taylor approximation to any arbitrary function as follows. Let

$$\alpha_i = h^*(z^o)$$

$$\alpha i \frac{\delta h^*(z^o)}{\delta z_{i-1}}, \; i = 2, \ldots, n+1 \quad \text{.......} \quad (6)$$

$$\alpha_j = \frac{\delta^2 h^*(z^o) i=1}{\delta zv \;\; \delta zm}, \; j = n+2, \ldots, 1/2(n+1)(n+2)$$

and

$$\begin{aligned} b_1(z^o) &= 1 \\ b_i(z^o) &= z_{i-1} - z_{i=1}, \; i=2, \ldots, n+1 \quad \text{.......} \quad (7) \\ b_j(z^o) &= 1/2(z_v - z^o{}_v)(z_m = z^o{}_m), \; j=n+2, \ldots, 1/2(n+1)(n+2) \end{aligned}$$

which, when combined with (4), yields the second-order Taylor series expansion. Forms that can be interpreted as a second-order Taylor series approximation to an arbitrary function are called **second-order numerical approximation** (see Chambers, 1988, Section 5.1).

[2] Output supply and input demand elasticities are calculated using formula :

a. cross-price elasticity :

$$e_{ij} = \frac{a_{ij}}{2 X_i} \left(\frac{p_j}{p_i}\right)^{1/2} \quad \text{for } i \# j$$

b. own-price elasticity :

$$e_{ij} = -e_{ij} - e_{ik} \quad \text{for } j \# k$$

where: a_{ij} is estimated parameter, X_i is variable quantity; an p is price.

Table A1. Selected econometric specifications that are estimated.

Number	Specification

I. Simultaneous Estimation of Profit Function, Yield Response, Fertilizer Demand, and Labor Demand :

$$\pi = -a_o p - a_1 w_1 - a_2 w_2 - 2a_{o1} p^{1/2} w_1^{1/2} - 2a_{o2} p^{1/2} w_2^{1/2}$$
$$-2a_{12} w_1^{1/2} w_2^{1/2} - a_{oz} p z - a_{1z} w_1 z - a_{2z} w_2 z + e\pi$$
$$Y = -a_o - a_{o1}(\frac{w_1}{p})^{1/2} - a_{o2}(\frac{w_2}{p})^{1/2} - a_{oz} z + e_y$$
$$F = a_1 + a_{o1}(\frac{p_1}{w_1})^{1/2} + a_{12}(\frac{w_2}{w_1})^{1/2} + a_{1z} z + e_f$$
$$L = a_2 + a_{o2}(\frac{p}{w_2})^{1/2} + a_{12}(\frac{w_1}{w_2})^{1/2} + a_{2z} z + e_i$$

II. Simultaneous Estimation of Yield Response, Fertilizer Demand, and Labor Demand :

$$Y = -a_o - a_{o1}(\frac{w_1}{p})^{1/2} - a_{o2}(\frac{w_2}{p})^{1/2} - a_{oz} z + e_y$$
$$F = a_1 + a_{o1}(\frac{p_1}{w_1})^{1/2} + a_{12}(\frac{w_2}{w_1})^{1/2} + a_{1z} z + e_f$$
$$L = a_2 + a_{o2}(\frac{p}{w_2})^{1/2} + a_{12}(\frac{w_1}{w_2})^{1/2} + a_{2z} z + e_i$$

III. Simultaneous Estimation of Profit Function, Fertilizer Demand, and Labor Demand :

$$\pi = -a_o p - a_1 w_1 - a_2 w_2 - 2a_{o1} p^{1/2} w_1^{1/2} - 2a_{o2} p^{1/2} w_2^{1/2}$$
$$-2a_{12} w_1^{1/2} w_2^{1/2} - a_{oz} p z - a_{1z} w_1 z - a_{2z} w_2 z + e$$
$$F = a_1 + a_{o1}(\frac{p_1}{w_1})^{1/2} + a_{12}(\frac{w_2}{w_1})^{1/2} + a_{1z} z + e_f$$
$$L = a_2 + a_{o2}(\frac{p}{w_2})^{1/2} + a_{12}(\frac{w_1}{w_2})^{1/2} + a_{2z} z + e_i$$

where: π is variable profit; Y is paddy production; F is fertilizer quantity; L is labor workdays; p is paddy price; w_i is fertilizer price; w_2 is labor wage rate; and z is time index (from 1 to T). All quantities are in per hectare basis.